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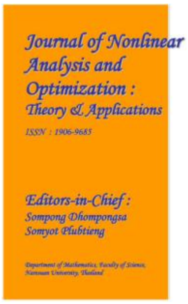

FUZZY G_T-FIRST CATEGORY AND FUZZY STRONGLY G_T-FIRST CATEGORY SETS.

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Abstract: In this paper we introduce a new concept of fuzzy $\mathbf{g_t}$-first category and fuzzy strongly $\mathbf{g_t}$-first category sets. Several properties are also discussed with suitable examples.
Keywords: Fuzzy closed sets, fuzzy locally closed sets, fuzzy $\mathbf{g_t}$-somewhere dense, fuzzy $\mathbf{g_t}$-nowhere dense sets, fuzzy $\mathbf{g_t}$-first category sets, fuzzy strongly $\mathbf{g_t}$-first category sets.

1. Introduction

In order to deal with uncertainties, the idea of fuzzy sets and fuzzy set operations was introduced by L. A. Zadeh [8]. The first notion of fuzzy topological space had been defined by C. L. Chang [2]. The fuzzy first and second category were introduced and studied by the authors Dr. G. Thangaraj and Dr. G. Balasubramanian [5]. The fuzzy strongly first category sets were introduced and studied by Dr. G. Thangaraj and R. Palani [6]. The fuzzy $\mathbf{g_t}$-nowhere dense set were introduced and studied by the authors Dr. S. Anjalmose and M. Kalaimathi [1]. In this paper we introduce a new class of fuzzy $\mathbf{g_t}$-first category and fuzzy strongly $\mathbf{g_t}$-first category sets. Several properties are also discussed with suitable examples.

2. Preliminaries

Definition 2.1: [3]
A fuzzy set η of a fuzzy topological space X is called fuzzy locally closed set if η = $(\gamma \wedge \zeta)$, where γ is a fuzzy open set and ζ is fuzzy closed set. The complement of fuzzy locally closed set is called fuzzy locally open set.
Definition 2.2: [4]
A fuzzy set η in a fuzzy topological space (X, T) is called fuzzy somewhere dense if $int\ cl\ (\eta) \neq 0$, in (X, T).
Definition 2.3: [7]
If η is a fuzzy somewhere dense set in a fuzzy topological space (X, T), then $1 - \eta$ is called a complement of fuzzy somewhere dense set in (X, T). It is to be denoted as fuzzy cs dense set in (X, T).
Definition 2.4: [1]
A fuzzy set η in a fuzzy topological space (X, T) is called fuzzy g_t-dense if there exists no fuzzy g_t-closed set β in (X, T) such that $\eta < \beta < 1$.
Definition 2.5: [1]
A fuzzy set η in a fuzzy topological space (X, T) is called fuzzy g_t-nowhere dense if there exists no non-zero fuzzy g_t-open set μ in (X, T) such that $\mu < g_t - cl(\eta)$. That is, $g_t - int\ g_t - cl(\eta) = 0$.

3. Fuzzy g_t-somewhere dense set.

Definition 3.1:
A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy g_t-somewhere dense set if $g_t - int\ g_t - cl(\lambda) \neq 0$ in (X, T).

Example 3.2:

Let $X = \{\lambda, \mu, \gamma\}$. The fuzzy sets λ, μ and γ are defined on X as follows:

$\lambda: X \to [0, 1]$ defined as $\lambda\left(\frac{a}{1}, \frac{b}{0.8}, \frac{c}{0.5}\right)$,

$\mu: X \to [0, 1]$ defined as $\mu\left(\frac{a}{0.9}, \frac{b}{0.8}, \frac{c}{0.3}\right)$,

$\gamma: X \to [0, 1]$ defined as $\gamma\left(\frac{a}{0.7}, \frac{b}{0.5}, \frac{c}{0.1}\right)$.

Then T = {0, λ, μ, γ, 1} is a fuzzy topology on X. Now the fuzzy sets $1 - \lambda, \alpha, \varepsilon, \delta, \zeta, \eta, \vartheta, \nu$ and $\sigma$are fuzzy locally closed in (X, T), since $\lambda \wedge (1 - \lambda) = 1 - \lambda, \lambda \wedge (1 - \mu) = \alpha$, $\lambda \wedge (1 - \gamma) = \varepsilon$, $\mu \wedge (1 - \lambda) = \delta$, $\mu \wedge (1 - \mu) = \zeta$, $\mu \wedge (1 - \gamma) = \eta$, $\gamma \wedge (1 - \lambda) = \vartheta$, $\gamma \wedge (1 - \mu) = \nu$and $\gamma \wedge (1 - \gamma) = \sigma$. Therefore the fuzzy sets $\lambda, 1 - \alpha, 1 - \varepsilon, 1 - \delta, 1 - \zeta, 1 - \eta, 1 - \vartheta, 1 - \nu$ and $1 - \sigma$ are fuzzy locally open sets. The fuzzy sets $1 - \lambda$, α, $\varepsilon, 1 - \mu$, ω, $1 - \gamma$are fuzzy gt-closed in (X, T), since $\lambda \wedge (1 - \lambda) = 1 - \lambda$, $\lambda \wedge (1 - \mu) = \alpha$, $\lambda \wedge (1 - \gamma) = \varepsilon$, $(1 - \alpha) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \alpha) \wedge (1 - \mu) = \alpha$, $(1 - \alpha) \wedge (1 - \gamma) = \varepsilon$, $(1 - \varepsilon) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \varepsilon) \wedge (1 - \mu) = \alpha$, $(1 - \varepsilon) \wedge (1 - \gamma) = \varepsilon, (1 - \delta) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \delta) \wedge (1 - \mu) = 1 - \mu$, $(1 - \delta) \wedge (1 - \gamma) = (0.3, 0.5, 0.7) = \omega\ (say), (1 - \zeta) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \zeta) \wedge (1 - \mu) = 1 - \mu, (1 - \zeta) \wedge (1 - \gamma) = \omega$, $(1 - \eta) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \eta) \wedge (1 - \mu) = 1 - \mu$, $(1 - \eta) \wedge (1 - \gamma) = \omega$, $(1 - \vartheta) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \vartheta) \wedge (1 - \mu) = 1 - \mu$, $(1 - \vartheta) \wedge (1 - \gamma) = 1 - \gamma$, $(1 - \nu) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \nu) \wedge (1 - \mu) = 1 - \mu, (1 - \nu) \wedge (1 - \gamma) = 1 - \gamma$, $(1 - \sigma) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \sigma) \wedge (1 - \mu) = 1 - \mu, (1 - \sigma) \wedge (1 - \gamma) = 1 - \gamma$, where the fuzzy sets $\lambda, 1 - \alpha, 1 - \varepsilon, 1 - \delta, 1 - \zeta, 1 - \eta, 1 - \vartheta, 1 - \nu$ and $1 - \sigma$ are fuzzy locally open sets in (X, T). Therefore the fuzzy sets λ, $1 - \alpha$, $1 - \varepsilon$, μ, $1 - \omega$, γ are fuzzy gt-open in (X, T). The fuzzy sets α, $\varepsilon, 1 - \mu$, ω, $1 - \gamma$, $1 - \alpha, 1 - \zeta$in (X, T) are fuzzy gt-somewhere dense. Since $g_t - intg_t - cl(\alpha) \neq 0$, $g_t - intg_t - cl(\varepsilon) \neq 0$, $g_t - intg_t - cl(1 - \mu) \neq 0$, $g_t - intg_t - cl(\omega) \neq 0$, $g_t - intg_t - cl(1 - \alpha) \neq 0$, $g_t - intg_t - cl(1 - \gamma) \neq 0$, $g_t - intg_t - cl(1 - \zeta) \neq 0$. The fuzzy set $1 - \lambda$ is not of fuzzy g_t-somewhere dense in (X, T), since $g_t - intg_t - cl(1 - \lambda) = 0$.

Proposition 3.3:

A fuzzy set λ is called fuzzy g_t-somewhere dense set in a fuzzy topological space (X, T), then λ is fuzzy locally closed in (X, T).

Proof:

In Example 3.2, the fuzzy set α, ε are fuzzy g_t-somewhere dense in a fuzzy topological space (X, T) also fuzzy locally closed in (X, T)

Proposition 3.4:

A fuzzy set λ is called fuzzy g_t-somewhere dense set in a fuzzy topological space (X, T), then λ is fuzzy closed in (X, T).

Proof:

In Example 3.2, the fuzzy set $1 - \mu, 1 - \gamma$are fuzzy g_t-somewhere dense in a fuzzy topological space (X, T) also fuzzy closed in (X, T).

Proposition 3.5:

A fuzzy set λ is fuzzy g_t-somewhere dense set in a fuzzy topological space (X, T), then λ is need not be fuzzy g_t-closed in (X, T).

Proof:

In Example 3.2, the fuzzy set $1 - \alpha, 1 - \zeta$are fuzzy g_t-somewhere dense in a fuzzy topological space (X, T) but not of fuzzy g_t-closed in (X, T).

Proposition 3.6:

A fuzzy set λ and μ are fuzzy g_t-somewhere dense sets in a fuzzy topological space (X, T), then $\lambda \wedge \mu$ is fuzzy g_t-somewhere dense in (X, T).

Proof:

Let λ and μ are fuzzy somewhere dense set in (X, T) then $g_t - intg_t - cl(\lambda) \neq 0$ and $g_t - intg_t - cl(\mu) \neq 0$. Now $g_t - intg_t - cl(\lambda \wedge \mu) = [g_t - intg_t - cl(\lambda)] \wedge [g_t - intg_t - cl(\mu)] \neq 0$. Hence $\lambda \wedge \mu$ is a fuzzy g_t-somewhere dense in a fuzzy topological space (X, T).

Proposition 3.7:

A fuzzy set λ be any non zero fuzzy set and μ be a fuzzy g_t-somewhere dense sets in a fuzzy topological space (X, T), then $\lambda \vee \mu$ is fuzzy g_t-somewhere dense in (X, T).

Proof:

Let λ be any non-zero fuzzy set and μ be a fuzzy g_t-somewhere dense set in (X, T) then $g_t - intg_t - cl(\mu) \neq 0$. Now $g_t - intg_t - cl(\lambda \vee \mu) = [g_t - intg_t - cl(\lambda)] \vee [g_t - intg_t - cl(\mu)] \neq 0$. Hence $\lambda \vee \mu$ is a fuzzy g_t-somewhere dense in a fuzzy topological space (X, T).

Proposition 3.8:

Let X and Y be fuzzy topological space (X, T) such that X is product related to Y. If λ is a fuzzy g_t-somewhere dense in X and $\mu$is a fuzzy g_t-somewhere dense in Y, then the product $\lambda \times \mu$ is a fuzzy g_t-somewhere dense in the product space $X \times Y$.

Proof:

Let λ be a fuzzy g_t-somewhere dense in X and μ be a fuzzy g_t-somewhere dense in Y. Then $g_t - int\ g_t - cl(\lambda) \neq 0$ in (X, T) and $g_t - int\ g_t - cl(\mu) \neq 0$ in (Y, S), since X is product related to Y.

Now $g_t - int\ g_t - cl(\lambda \times \mu) = g_t - int\ [g_t - cl(\lambda) \times g_t - cl(\mu)]$

$$= g_t - int\ g_t - cl(\lambda) \times g_t - int\ g_t - cl(\mu)$$

$\neq 0$.

Thus the product $\lambda \times \mu$ is a fuzzy g_t-somewhere dense in the product space $X \times Y$.

Proposition 3.9:

A fuzzy set λ is a fuzzy g_t-somewhere dense set in a fuzzy topological space (X, T), then λ need not be fuzzy g_t-open in (X, T).

Proof:

In Example 3.2, the fuzzy set $\alpha, \varepsilon$are fuzzy g_t-somewhere dense in a fuzzy topological space (X, T) but not of fuzzy g_t-open in (X, T).

Proposition 3.10:

A fuzzy set λ and μ are fuzzy g_t-somewhere dense sets in a fuzzy topological space (X, T), then $\lambda \vee \mu$ is fuzzy g_t-somewhere dense in (X, T).

Proposition 3.11:

A fuzzy set λ is fuzzy g_t-closed with $g_t - int(\lambda) \neq 0$ then in a fuzzy topological space (X, T), then λ is a fuzzy g_t-somewhere dense in (X, T).

Proposition 3.12:

A fuzzy set λ is fuzzy g_t-somewhere dense in a fuzzy topological space (X, T), then $g_t - cl(\lambda)$is a fuzzy g_t-somewhere dense in (X, T).

Proposition 3.13:

A fuzzy set λ is fuzzy g_t-somewhere dense in a fuzzy topological space (X, T), then $g_t - cl\ g_t - int(1 - \lambda) \neq 1$.

4. Fuzzy g_t-cs dense sets.

Definition 4.1:

If λ is a fuzzy g_t-somewhere dense set in a fuzzy topological space (X, T), then $1 - \lambda$ is called a complement of fuzzy g_t-somewhere dense set in (X, T). It is to be denoted as fuzzy g_t-cs dense set in (X, T).

In example 3.2, the fuzzy sets $\alpha, \varepsilon, 1 - \mu, \omega, 1 - \gamma, 1 - \zeta, 1 - \nu,$ are fuzzy g_t-cs dense in (X, T), since $1 - \alpha,\ 1 - \varepsilon,\ \mu, 1 - \omega,\ \gamma, \zeta, \nu,$ are fuzzy g_t-somewhere dense in (X, T).

Proposition 4.2:

A fuzzy set λ be a fuzzy g_t-cs dense sets in a fuzzy topological space (X, T), then $g_t - int(\lambda)$ is not a fuzzy g_t-dense in (X, T).

Proof:

Let λ be fuzzy g_t-cs dense set in (X, T) then $1 - \lambda$ is a fuzzy g_t-somewhere dense in (X, T). Therefore $g_t - intg_t - cl(1 - \lambda) \neq 0$ implies that $1 - [g_t - clg_t - int(\lambda)] \neq 0$ implies that $g_t - cl\ g_t - int(\lambda) \neq 1$. Hence $g_t - int(\lambda)$ is not a fuzzy g_t-dense in (X, T).

Proposition 4.3:

A fuzzy set λ and μ be a fuzzy g_t-cs dense sets in a fuzzy topological space (X, T), then $(\lambda \wedge \mu)$ is fuzzy g_t-cs dense in (X, T).

Proposition 4.4:

A fuzzy set λ be a fuzzy g_t-cs dense sets in a fuzzy topological space (X, T), then $g_t - cl\, g_t - int(\lambda) \neq 1$.

5. Fuzzy g_t-first category and fuzzy strongly gt-first category set.

Definition 5.1:

Let (X, T) be a fuzzy topological space. A fuzzy set λ in X is called a fuzzy strongly g_t-nowhere dense set, if $\lambda \wedge (1 - \lambda)$ is a fuzzy g_t-nowhere dense set in (X, T). That is., λ is a fuzzy strongly g_t-nowhere dense set in (X, T), if $g_t - int\, g_t - cl[\lambda \wedge (1 - \lambda)] = 0$ in (X, T).

Example 5.2:

Let $X = \{\lambda, \mu\}$. The fuzzy sets λ and μ are defined on X as follows:

$\lambda: X \to [0, 1]$ defined as $\lambda\left(\frac{a}{0.2}, \frac{b}{0.7}\right)$,

$\mu: X \to [0, 1]$ defined as $\mu\left(\frac{a}{0.1}, \frac{b}{0.5}\right)$.

Then T = { $0, \lambda, \mu, 1$} is a fuzzy topology on X. Now the fuzzy sets μ, α, β and $\gamma$are fuzzy locally closed in (X, T), since $\lambda \wedge (1 - \lambda) = \alpha, \mu \wedge (1 - \lambda) = \gamma$, $\lambda \wedge (1 - \mu) = \beta$ and $\mu \wedge (1 - \mu) = \mu$. Then $1 - \mu, 1 - \alpha, 1 - \beta$ and $1 - \gamma$ are fuzzy locally open sets in (X, T). The fuzzy sets $1 - \lambda, 1 - \mu$ and $1 - \beta$ are fuzzy gt-closed, since $(1 - \alpha) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \alpha) \wedge (1 - \mu) = 1 - \beta$, $(1 - \beta) \wedge (1 - \lambda) = 1 - \lambda, (1 - \beta) \wedge (1 - \mu) = 1 - \beta$, $(1 - \gamma) \wedge (1 - \lambda) = 1 - \lambda$, $(1 - \gamma) \wedge (1 - \mu) = 1 - \mu, (1 - \mu) \wedge (1 - \lambda) = 1 - \lambda, (1 - \mu) \wedge (1 - \mu) = 1 - \mu$, where $1 - \mu, 1 - \alpha, 1 - \beta$ and $1 - \gamma$ are fuzzy locally open sets in (X, T). Therefore the fuzzy sets λ, μ and β in (X, T) are fuzzy g_t-open.

Now consider the fuzzy sets τ and ω as defined on x as follows

$\tau : X \to [0, 1]$ defined as $\tau\left(\frac{a}{0.7}, \frac{b}{0.2}\right)$,

$\omega: X \to [0, 1]$ defined as $\omega\left(\frac{a}{0.6}, \frac{b}{0.3}\right)$.

The fuzzy sets τ, ω, λ are fuzzy strongly g_t-nowhere dense in (X, T), since $g_t - int\, g_t - cl[\tau \wedge (1 - \tau)] = g_t - int\, g_t - cl(0.3, 0.2) = 0$, $g_t - int\, g_t - cl[\omega \wedge (1 - \omega)] = g_t - int\, g_t - cl(0.4, 0.3) = 0, g_t - int\, g_t - cl[\lambda \wedge (1 - \lambda)] = g_t - int\, g_t - cl(0.2, 0.3) = 0$. But μ, β are not of fuzzy strongly g_t-nowhere dense in (X, T), since $g_t - int\, g_t - cl[\mu \wedge (1 - \mu)] = g_t - int\, g_t - cl(\mu) \neq 0$, $g_t - int\, g_t - cl[\beta \wedge (1 - \beta)] = g_t - int\, g_t - cl(\beta) \neq 0$.

Definition 5.3:

A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy g_t-first category set if λ = $V_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy g_t-nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be fuzzy g_t-second category.

In example 5.2, The fuzzy sets $1 - \lambda$, τ and ω are fuzzy g_t-nowhere dense sets, since $g_t - int g_t - cl(1 - \lambda) = 0$, $g_t - int g_t - cl(\tau) = 0$, $g_t - int g_t - cl(\omega) = 0$. Therefore the union of fuzzy g_t-nowhere dense sets $1 - \lambda$, τ and ω is $1 - \lambda$, that is $[(1 - \lambda) \vee \tau \vee \omega] = 1 - \lambda$. Hence $1 - \lambda$ is a fuzzy g_t-first category set in (X, T). Otherwise some of the fuzzy sets λ, μ, β, $1 - \mu, 1 - \beta$ are fuzzy g_t-second category in (X, T).

Definition 5.4:

A fuzzy set λin a fuzzy topological space (X, T) is called fuzzy g_t-residual set if $1 - \lambda$ is fuzzy g_t-first category in (X, T).

In Example 5.2, $1 - \lambda$ is a fuzzy g_t-first category set in (X, T), since the union of fuzzy g_t-nowhere dense sets $1 - \lambda$, τ and ω are $1 - \lambda$, that is $[(1 - \lambda) \vee \tau \vee \omega] = 1 - \lambda$. Hence λis a fuzzy g_t-residual in (X, T).

Definition 5.5:

A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy strongly g_t-first category set if $\lambda = V_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy strongly g_t-nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be fuzzy strongly g_t-second category.

In Example 5.2, $(\tau \vee \omega \vee \lambda) = (0.7, 0.7) = \delta$ (say) is a fuzzy strongly g_t-first category in (X, T), since τ, ω, λ are fuzzy strongly g_t-nowhere dense sets in (X, T).

Definition 5.6:

A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy strongly g_t-residual set if $1 - \lambda$ is fuzzy strongly g_t-first category in (X, T).

In Example 5.2, $(\tau \vee \omega \vee \lambda) = (0.7, 0.7) = \delta$ (say) is a fuzzy strongly g_t-first category in (X, T), where τ, ω, λ are fuzzy strongly g_t-nowhere dense sets in (X, T).

Proposition 5.7:

If λ is a fuzzy g_t-nowhere dense set in a fuzzy topological space (X, T), then λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proof:

Let λ be a fuzzy g_t-nowhere dense set in (X, T). Then $g_t - int\ g_t - cl(\lambda) = 0$, in (X, T). Since $\lambda \wedge (1 - \lambda) \leq \lambda$ in (X, T), $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] \leq g_t - int\ g_t - cl(\lambda)$ and hence $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] \leq 0$. That is $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] = 0$. Hence λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Remark 5.8.

A fuzzy strongly g_t-nowhere dense set in a fuzzy topological space (X, T) need not be a fuzzy g_t-nowhere dense set in (X, T).

In example 5.2, λ is a fuzzy strongly g_t-nowhere dense set, but not a fuzzy g_t-nowhere dense set in (X, T).

Proposition 5.9:

If $g_t - int(\lambda)$ is a fuzzy g_t-dense set, for a fuzzy set λ defined on X, in a fuzzy topological space (X, T), then λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proof:

Suppose that $g_t - int(\lambda)$ is a fuzzy g_t-dense set in (X, T). Then $gt - g_t - cl[g_t - int(\lambda)] = 1$ in (X, T) and $1 - [g_t - cl\ g_t - int(\lambda)] = 0$. This implies that $g_t - int\ g_t - cl(1 - \lambda) = 0$ in (X, T). Since $\lambda \wedge (1 - \lambda) \leq 1 - \lambda$, $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] \leq g_t - int\ g_t - cl(1 - \lambda)$ and hence $g_t - int\ g_t - cl(1 - \lambda) \leq 0$. That is., $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] = 0$. Hence λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proposition 5.10:

If $1 - \lambda$ is a fuzzy g_t-nowhere dense set in a fuzzy topological space (X, T), then λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proof:

Suppose that $1 - \lambda$ is a fuzzy g_t-nowhere dense set in (X, T). Then, $g_t - int\ g_t - cl(1 - \lambda) = 0$ in (X, T). Since $\lambda \wedge (1 - \lambda) \leq 1 - \lambda$, $[g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] \leq g_t - int\ [g_t - cl(1 - \lambda)]$ and hence $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] \leq 0$. That is., $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] = 0$ in (X, T) and hence λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proposition 5.11:

If $g_t - cl\ g_t - int(1 - \lambda) = 1$, for a fuzzy set λ defined on X in a fuzzy topological space (X, T), then λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proof:

Suppose that $g_t - cl\ g_t - int(1 - \lambda) = 1$ in (X, T). Then $1 - [g_t - cl\ g_t - int(1 - \lambda) = 0$ and $1 - \{1 - [g_t - int\ g_t - cl(\lambda)]\} = 0$. This implies that $g_t - int\ g_t - cl(\lambda) = 0$ in (X, T). Thus λ is a fuzzy g_t-nowhere dense set in (X, T). Then, by proposition 5.7, λ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proposition 5.12:

If λ is a fuzzy strongly g_t-nowhere dense set in a fuzzy topological space (X, T), then $1-\lambda$ is also a fuzzy strongly g_t-nowhere dense set in (X, T).

Proof:

Let λ be a fuzzy strongly g_t-nowhere dense set in (X, T). Then $g_t - int\ g_t - cl[\lambda \wedge (1 - \lambda)] = 0$ in (X, T). Now $g_t - int\ g_t - cl\{(1-\lambda) \wedge [1-(1 - \lambda)]\} = g_t - int\ g_t - cl[(1-\lambda) \wedge \lambda]$ and hence $g_t - int\ g_t - cl\{(1-\lambda) \wedge [1-(1 - \lambda)]\} = 0$. This implies that $1-\lambda$ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proposition 5.13:

If λ is a fuzzy g_t-nowhere dense set in a fuzzy topological space (X, T), then $1-\lambda$ is a fuzzy strongly g_t-nowhere dense set in (X, T).

Proof:

Let λ be a fuzzy g_t-nowhere dense set in (X, T). Then, by proposition 5.7, λ is a fuzzy strongly g_t-nowhere dense set in (X, T), and by proposition 5.12, $1-\lambda$ is a fuzzy strongly g_t-nowhere dense set in(X, T).

Proposition 5.14:

If λ is a fuzzy g_t-first category set in a fuzzy topological space (X, T), then λ is a fuzzy strongly g_t-first category set in (X, T).

Proof:

Let λ be a fuzzy g_t-first category set in (X, T). Then $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'$s are fuzzy g_t-nowhere dense sets in (X, T). By proposition 5.7, the fuzzy g_t-nowhere dense sets $(\lambda_i)'$s are fuzzy strongly g_t-nowhere dense sets in (X, T) and hence $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $(\lambda_i)'$ s are fuzzy strongly g_t-nowhere dense sets in (X, T), implies that λ is a fuzzy strongly g_t-first category set in (X, T).

Remark 5.15:

The converse of the above proposition need not be true. A fuzzy strongly g_t-first category set need not be a fuzzy g_t-first category set in a fuzzy topological space.

For, in example 5.2, $(\tau \vee \omega \vee \lambda) = \delta$ is a fuzzy strongly g_t-first category set in (X, T) but not a fuzzy g_t-first category set in (X, T).

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